Universidade Estadual Paulista
"Júlio de Mesquita Filho"

Exercícios Resolvidos - 14/05/2016

Cálculo 3 - Ciências da Computação

Professor:
Vinícius F. Wasques
viniwasques@hotmail.com

17 de maio de 2016

1 Exercícios :

Exercício 1.1. *Calcule* $\frac{dz}{dt}$ *pelos dois processos descritos em aula:*

(a) $z = sen(xy)$, $x = 3t$ *e* $y = t^2$

(b) $z = x^2 + 3y^2$, $x = sen(t)$ *e* $y = cos(t)$

(c) $z = ln(1 + x^2 + y^2)$, $x = sen(3t)$ *e* $y = cos(3t)$

Solução:

(a) $z = sen(xy)$, $x = 3t$ *e* $y = t^2$

Através do gradiente:

$\nabla f(\gamma(t)) = (t^2cos(3t^3), 3tcos(3t^3))$

$(\gamma(t))' = (3, 2t)$

$$\Rightarrow \frac{dz}{dt} = \nabla f(\gamma(t)).(\gamma(t))' = (t^2cos(3t^3), 3tcos(3t^3)).(3, 2t)$$

$$\Rightarrow \frac{dz}{dt} = 3t^2cos(3t^3) + 6t^2cos(3t^3) = 9t^2cos(3t^3)$$

Através da fórmula:

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}(\gamma(t))\frac{dx}{dt} + \frac{\partial z}{\partial y}(\gamma(t))\frac{dy}{dt}$$

$$\Rightarrow \frac{dz}{dt} = t^2cos(3t^3)3t + 3tcos(3t^3)2t$$

$$\Rightarrow \frac{dz}{dt} = 3t^2cos(3t^3) + 6t^2cos(3t^3) = 9t^2cos(3t^3)$$

(b) $z = x^2 + 3y^2$, $x = sen(t)$ *e* $y = cos(t)$

Através do gradiente:

$\nabla f(\gamma(t)) = (2sen(t), 6cos(t))$

$(\gamma(t))' = (cos(t), -sen(t))$

$$\Rightarrow \frac{dz}{dt} = \nabla f(\gamma(t)).(\gamma(t))' = (2sen(t), 6cos(t)).(cos(t), -sen(t))$$

$$\Rightarrow \frac{dz}{dt} = 2sen(t)cos(t) - 6cos(t)sen(t) = -4cos(t)sen(t)$$

Através da fórmula:

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}(\gamma(t))\frac{dx}{dt} + \frac{\partial z}{\partial y}(\gamma(t))\frac{dy}{dt}$$

$$\Rightarrow \frac{dz}{dt} = 2sen(t)cos(t) - 6cos(t)sen(t)$$

$$\Rightarrow \frac{dz}{dt} = -4cos(t)sen(t)$$

(c) $z = ln(1 + x^2 + y^2)$, $x = sen(3t)$ *e* $y = cos(3t)$

Através do gradiente:

$\nabla f(\gamma(t)) = (\frac{2sen(3t)}{1+sen^2(3t)+cos^2(3t)}, \frac{2cos(3t)}{1+sen^2(3t)+cos^2(3t)})$

$(\gamma(t))' = (3cos(3t), -3sen(3t))$

$$\Rightarrow \frac{dz}{dt} = \nabla f(\gamma(t)).(\gamma(t))'$$

$$= \left(\frac{2sen(3t)}{1 + sen^2(3t) + cos^2(3t)}, \frac{2cos(3t)}{1 + sen^2(3t) + cos^2(3t)}\right).(3cos(3t), -3sen(3t))$$

$$\Rightarrow \frac{dz}{dt} = \frac{6sen(3t)cos(3t)}{1 + sen^2(3t) + cos^2(3t)} + \frac{-6sen(3t)cos(3t)}{1 + sen^2(3t) + cos^2(3t)} = 0$$

Através da fórmula:

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}(\gamma(t))\frac{dx}{dt} + \frac{\partial z}{\partial y}(\gamma(t))\frac{dy}{dt}$$

$$\Rightarrow \frac{dz}{dt} = \frac{6sen(3t)cos(3t)}{1 + sen^2(3t) + cos^2(3t)} + \frac{-6sen(3t)cos(3t)}{1 + sen^2(3t) + cos^2(3t)}$$

$$\Rightarrow \frac{dz}{dt} = 0$$

Exercício 1.2. *Seja* $z = f(u - v, v - u)$. *Mostre que*

$$\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = 0$$

Solução:

Através da regra da cadeia temos:

$$\frac{\partial z}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial u} = \frac{\partial f}{\partial x}1 + \frac{\partial f}{\partial y}(-1)$$

$$\frac{\partial z}{\partial u} = \frac{\partial f}{\partial x} - \frac{\partial f}{\partial y}$$

Por outro lado,

$$\frac{\partial z}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v}$$

$$\frac{\partial z}{\partial v} = \frac{\partial f}{\partial x}(-1) + \frac{\partial f}{\partial y}1$$

$$\frac{\partial z}{\partial v} = -\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}$$

Desse modo,

$$\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = 0$$

Exercício 1.3. *Seja* $z = g(x, y)$, *uma função diferenciável dada implicitamente pela equação* $f(x, y, z) = 0$ *onde* f *é uma função diferenciável.*

(a) *Determine* $\frac{\partial z}{\partial y}$

(b) *A partir do item (a) determine* $\frac{\partial z}{\partial y}$ *onde* z *é dada implicitamente pela equação*

$$xy - y^2x + zxy - 2zy^2 = 4$$

Solução:

(a)

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial x}\frac{dx}{dy} + \frac{\partial f}{\partial y}\frac{dy}{dy} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial y} = 0$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial x}0 + \frac{\partial f}{\partial y}1 + \frac{\partial f}{\partial z}\frac{\partial z}{\partial y} = 0$$

$$\Rightarrow \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial y} = 0$$

$$\Rightarrow \frac{\partial z}{\partial y} = -\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial z}}$$

com $\frac{\partial f}{\partial z} \neq 0$

(b) *Como*

$$\frac{\partial z}{\partial y} = -\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial z}}$$

e

$$\frac{\partial f}{\partial y} = x - 2xy + zx - 4zy$$

$$\frac{\partial f}{\partial z} = xy - 2y^2$$

Então,

$$\frac{\partial z}{\partial y} = -\frac{x - 2xy + zx - 4zy}{xy - 2y^2}$$